Quantum Physics, Course KFY/7KVAF

WS 2020/2021

Seminar 5: Hydrogen atom, spin

1. Show that the first few hydrogenic atomic wavefunctions are orthonormal; in other words, calculate $\langle\psi_{100}|\psi_{100}\rangle$, $\langle\psi_{200}|\psi_{200}\rangle$, $\langle\psi_{100}|\psi_{200}\rangle$ etc. using $\psi_{100} = \frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} e^{-\rho}$, $\psi_{200} = \frac{1}{4\sqrt{2\pi}}\left(\frac{Z}{a_0}\right)^{3/2}(2-\rho)e^{-\rho/2}$ etc., where $\rho = Zr/a_0$, Z is nuclear charge and a_0 is the Bohr radius.

2. Calculate probability that a hydrogen $1s$ electron will be found within a distance $2a_0$ from the nucleus.

3. Calculate averages $\langle r\rangle$, $\langle r^2\rangle$ and most probable distance r_0 from the nucleus for the ground-state electron ($1s$ state).

4. Prove that the average value of r in the $1s$ and $2s$ states for a hydrogen-like atom is $3a_0/2Z$ and $6a_0/Z$, respectively.

5. What is the degeneracy of each of the hydrogen atomic energy levels (neglecting spin-orbit interaction)?

6. Proof that Pauli matrices σ_x, σ_y and σ_z obey relations
a) $[\sigma_x, \sigma_y]=2i\sigma_z$, $[\sigma_y, \sigma_z]=2i\sigma_x$, $[\sigma_z, \sigma_x]=2i\sigma_y$ and
b) $\sigma_x^2 = 1$, $\sigma_y^2 = 1$ a $\sigma_z^2 = 1$.

7. Calculate
a) anticommutator of Pauli matrices,
b) the eigenvalues and corresponding eigenvectors of the Pauli matrix σ_x and
c) proof the validity of equation $\sigma_x\sigma_y\sigma_z = i\mathbb{1}$.