Quantum Physics, Course KFY/7KVAF
WS 2020/2021
Seminar 6: Stationary perturbation theory

1. An electron is constrained to move on a line segment. Find a correction to energy levels given by small additional perturbation, which is an electrostatic field with intensity $\vec{E}$. What is the change of photon energy emitted when electron transition from first excited to ground state occur? [Hint: Consider charged particle on a line segment $0 \leq x \leq L$ and perturbation $V(x) = -eEx$. Use the first order perturbational theory on the particle in a box.]

2. Calculate the first-order correction to the energy of a particle constrained to move within the region $0 \leq x \leq a$ in the potential $V(x) = V_0 x$ for $0 \leq x \leq a/2$ and $V(x) = V_0(a - x)$ for $a/2 \leq x \leq a$, where V_0 is a constant.

3. Electron spin is in a strong magnetic field $\vec{B_0}$ in z direction and a weak magnetic field $\vec{b}$ in x direction is added. Find eigenvalues and corresponding eigenvectors a) exactly and subsequently b) using perturbation theory up to second order and copare both results. [Hint: Hamiltonian is given as $\hat{H} = \frac{e\hbar}{2m}\hat{\vec{\sigma}} \cdot \vec{B}$ (magnetic moment energy), where $\vec{B} = \vec{B_0} + \vec{b}$, $\hat{\vec{\sigma}}$ is a vector composed of Pauli matrices and $e < 0$.

4. Use first-order perturbation theory to calculate the first-order correction to the ground-state energy of a quartic oscillator whose potential energy is $V(x) = cx^4$. In this case, use a harmonic oscillator as the unperturbed system. What is the perturbing potential?

5. Let 1D harmonic oscillator is charged (charge e) and located in the electrostatic field $\vec{E}$. Calculate the change of ground state energy using the first-order and second-order perturbation theory. Found exact solution too and compare it with the previous approximated one. [Hint: The Hamiltonian corresponding to the perturbation for oscillations in x direction is given as $\hat{H}' = -eE\hat{x}$. Use the equation $\int_{-\infty}^{\infty} \psi_n^*(x) x \psi_m(x) dx = 0$ for $|m - n| \geq 2$ valid for eigenfunctions of oscillator. Remind ground-state and first excited-state eigenfunction: $\psi_0(x) = \sqrt{\frac{2}{x_0\sqrt{\pi}}}\exp(-\frac{x^2}{2x_0^2})$, $\psi_1(x) = \sqrt{\frac{2}{x_0\sqrt{\pi}}}\frac{x}{x_0}\exp(-\frac{x^2}{2x_0^2})$, where $x_0 = \sqrt{\hbar/m\omega}$.]