

# Quantum Physics, Course KFY/7KVAF

## WS 2022/2023

### Theme 2: Properties of linear operators

- Calculate commutator
  - $[\hat{A}\hat{B}, \hat{C}]$  of three operators  $\hat{A}, \hat{B}, \hat{C}$  using  $[\hat{A}, \hat{C}]$  and  $[\hat{B}, \hat{C}]$ ,
  - $[\frac{d}{dx}, x]$ ,
  - of position  $\hat{x}$  and momentum  $\hat{p}$  operators,
  - $[\hat{x} - \hat{p}, \hat{p} - \hat{x}]$ ,
  - $[\hat{x}\hat{p}, \hat{x}]$ ,
  - $[\frac{\partial}{\partial x}, f(x, y, z)]$ , where  $f$  is complex function,
  - $[\hat{x}, \Delta]$ .
- Which of the following  $\hat{A}$  operators are linear ( $u \equiv u(x)$  is complex function)?
  - $\hat{A}u = \lambda u$ , where  $\lambda$  is complex constant,
  - $\hat{A}u = u^*$ ,
  - $\hat{A}u = u^2$ ,
  - $\hat{A}u = \frac{du}{dx}$ ,
  - $\hat{A}u = \frac{1}{u}$ ,
  - $\hat{A}u = \frac{d^2u}{dx^2} + 3u^2$ .
- Find adjoint operators to operators a) – e) and check possible self-adjoint operators (Hermitian operator):
  - $\hat{A} = x$ ,
  - $\hat{A} = \frac{d}{dx}$ ,
  - $\hat{A} = f(x)$ ,  $f(x)$  is complex function,
  - $\hat{A} = f(x)$ ,  $f(x)$  is real function,
  - $\hat{A} = c$ ,  $c$  is complex number.
- Check if the following operators are Hermitian
  - the momentum operator  $\hat{p} = -i\hbar\nabla$ ,
  - the Laplace operator  $\Delta$ .
- Find the eigenvalues and corresponding eigenvectors of the following matrices:
  - $\begin{pmatrix} -1 & 2 \\ 2 & 2 \end{pmatrix}$
  - $\begin{pmatrix} -2 & 0 & 0 \\ 0 & -1 & 2 \\ 0 & 2 & 2 \end{pmatrix}$
  - two-level Hamiltonian  $\hat{H} = \begin{pmatrix} E_0 & A \\ A & E_0 \end{pmatrix}$ ,  $E_0, A \in \mathbb{R}$ .
- Find the eigenvalues and corresponding eigenvectors of the following operators:
  - $\frac{d}{dx}$ ,
  - $i(\frac{d}{dx})$ ,
  - $x + \frac{d}{dx}$ ,
  - $-i(\frac{d}{d\varphi})$ , where  $\varphi$  is rotation by angle around the axis  $z$  (spherical coordinate).
- Match the eigenfunctions in right column to their operators in left column. What is the eigenvalue for each eigenfunction?
  - $(1 - x^2)\frac{d^2}{dx^2} - x\frac{d}{dx}$  a.  $4x^4 - 12x^2 + 3$
  - $\frac{d^2}{dx^2}$  b.  $5x^4$
  - $x\frac{d}{dx}$  c.  $e^{3x} + e^{-3x}$
  - $\frac{d}{dx^2} - 2x\frac{d}{dx}$  d.  $x^2 - 4x + 2$
  - $x\frac{d^2}{dx^2} + (1 - x)\frac{d}{dx}$  e.  $4x^3 - 3x$