Quantum Physics, Course KFY/7KVAF

WS 2022/2023

Theme 3: Simple quantum systems

1. Show, that the total momentum operator $\hat{\boldsymbol{P}} = \sum_i \hat{\boldsymbol{p}}_i$ for a set of particles is conserved (constant) if no external forces are acting.

2. Show the probability current (flux) for
a) free particle described by plane wave $\psi(x) = Ae^{Et-\sqrt{2mE}x/(i\hbar)}$,
b) wave function $\psi(x) = Ae^{Et-\sqrt{2mE}x/(i\hbar)} + Be^{Et+\sqrt{2mE}x/(i\hbar)}$.

3. One-particle wave function is $\psi(x,0) = Ne^{-x^2/(2a^2)+ik_0x}$ at time $t = 0$ (real numbers a and k_0). Determine (normalization) coefficient N, typical size of particle localisation and the probability current.

4. Calculate the expected value of coordinate $\langle\hat{x}\rangle$ and momentum $\langle\hat{p}\rangle$ of the particle from the previous exercise.

5. A rigid body described by moment of inertia J is rotating around free axis z (rigid rotator). Find its eigenstates (wave functions) and eigenenergies. [Hint: Consider polar coordinates (r, φ). You should obtain Hamiltonian $\hat{H} = \frac{\hat{L}_z^2}{2J}$ with $\hat{L}_z = -i\hbar\frac{\partial}{\partial\varphi}$.]

6. Determine the expected values of a) the orbital angular momentum $\hat{L}_z$ and b) the square of the orbital angular momentum $\hat{L}_z^2$ from the previous exercise.

7. Find the reflection and transmission coefficients for the one-dimensional potential step if the particles are incident from the right and the potential is defined as $V(x) = 0$ for $x < 0$ and $V(x) = v_0$ for $x > 0$.