Quantum Physics, Course KFY/7KVAF

WS 2022/2023

Topic 4: Angular momentum

1. Proof that the orbital angular momentum operator $\hat{\boldsymbol{L}} = \hat{\boldsymbol{r}} \times \hat{\boldsymbol{p}}$ is Hermitian operator.

2. Show that $\hat{\boldsymbol{L}} \times \hat{\boldsymbol{L}} = i\hbar\hat{\boldsymbol{L}}$.

3. Show that the square of the magnitude of the orbital angular momentum $\hat{\boldsymbol{L}}^2 = \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2$ commute with all $\hat{L}_j$ $(j = x, y, z)$.

4. Calculate commutator of the following operators:
a) components of $\hat{\boldsymbol{L}}$ and position $\hat{x}$,
b) components of $\hat{\boldsymbol{L}}$ and a momentum $\hat{p}_x$ operator.

5. Examine the operators below along with an appropriate given function. Determine if the given function is simultaneously an eigenfunction of both operators. Is this what you expected?
a) $\hat{L}_z$, $\hat{\boldsymbol{L}}^2$ with function $Y_{00}(\theta, \phi) = \frac{1}{\sqrt{4\pi}}$,
b) $\hat{L}_x$, $\hat{L}_z$ with function $Y_{00}(\theta, \phi) = \frac{1}{\sqrt{4\pi}}$,
c) $\hat{L}_z$, $\hat{\boldsymbol{L}}^2$ with function $Y_{10}(\theta, \phi) = \sqrt{\frac{3}{4\pi}} \cos\theta$,
d) $\hat{L}_x$, $\hat{L}_z$ with function $Y_{10}(\theta, \phi) = \sqrt{\frac{3}{4\pi}} \cos\theta$.
[Hint: You can use the general angular momentum relationships:
$J^2 Y_{jm} = \hbar^2(j(j+1))Y_{jm}$ and $J_z Y_{jm} = \hbar m Y_{jm}$ together with the information
$\hat{L}_x Y_{l,m} = \frac{1}{2}(\hat{L}_+ Y_{l,m} + \hat{L}_- Y_{l,m}) = \frac{1}{2}\sqrt{l(l+1) - m(m+1)}\hbar Y_{l,m+1} + \frac{1}{2}\sqrt{l(l+1) - m(m-1)}\hbar Y_{l,m-1}$]