Quantum Physics, Course KFY/7KVAF

WS 2022/2023

Theme 8: Non-stationary perturbation theory

1. A particle is in a one-dimensional harmonic oscillator potential $V(x) = \frac{1}{2}kx^2$. It is initially in its ground state. The spring constant k is suddenly doubled, and the particle's energy is then measured. What is the probability of finding that particle in the ground state of the new potential? [Hint: Consider ground state wave functions of linear harmonic oscillator $\psi_0(x) = (\frac{m\omega}{\hbar\pi})^{\frac{1}{4}} \exp(-\frac{m\omega x^2}{2\hbar})$ with spring constant k and frequency ω. New spring constant $k' = 2k$ and frequency $\omega' = \sqrt{2}\omega$ belong to the wave function $\psi_0'(x)$. The probability of sudden transition is $w_0 = |\langle\psi_0'(x)\psi_0(x)\rangle|^2$.]

2. A particle is in its ground state in a box with infinite walls at $x = 0$ and $x = L$. The wall of the box at $x = L$ is suddenly moved to $x = 2L$. Calculate the probability that the particle will be found in the ground state of the expanded box.

3. The hydrogen nucleus with the mass number $Z = 3$ (tritium ^{3}H) is not stable and decays into ^{3}He$^+$ (by β-decay). Let's assume that ${}^3_1\mathrm{H} \rightarrow {}^3_2\mathrm{He}^+ + \mathrm{e}^- + \bar{\nu}$ decay is extremely quick, just charge is enlarged, and the original electron is not influenced (at $t = 0$, the ground state of ^{3}H is expected). Calculate the probability of the resulting excited state of He$^+$.